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Norwegian University of Science and Technology

Introduction to Hyper-heuristics

General purpose solvers

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Outline

Optimisation problems are common

Motivation

Here's a couple of combinatorial and parametric optimisation problems for you:

1. Pack a selection of items inside a container with limited capacity, such that the total value of the items is maximal.
2. Assign guests to seats in order to avoid conflicting scenarios at a gala dinner.
3. Visit all REMA 1000s and Bunnpris stores in the city such that the distance travelled is minimal.
4. Fine tune hyperparameters of your machine learning model.

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Here's a couple of combinatorial and parametric optimisation problems for you:

1. Pack a selection of items inside a container with limited capacity, such that the total **value** of the items is **maximal**.
2. **Assign** guests to seats in order to avoid **conflicting** scenarios at a gala dinner.
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Optimisation problems are hard

Motivation

- **Knapsack Problems**: NP-hard with complexity of $\mathcal{O}(cn)$ via dynamic programming. Intractable when $c \gg n$.
- **Constraint Satisfaction Problems (CSP)**: NP-complete with upper-bound of $\mathcal{O}(d^n)$ for a maximum domain size d and n variables.
- **Vehicle Routing**: NP-complete with upper-bound of $\mathcal{O}(n!)$ for n visited nodes.
- **Parametric optimisation**: Can be transformed into a CSP if considering discrete domains so $\mathcal{O}(d^n)$

Let's try to solve the seating problem!

Heuristics

Three available tables (one with 5 seats and two with 6 seats each). Here are the hard requirements:

- The *Pharaoh's family* (Dad, Mom and two kids) can't be near the Priest. The family kids are intrigued by the Robot so they want to sit on the same table.
- Demis, Vangelis and Mikis (*The Greek*) need to be on the same table.
- Wilson and Akerfelt want to be seated with *The Greek*, but don't want to be in the same table than Parsons.
- Parsons and Gilmour prefer to be with one another.
- Dickinson, Harris, Dio and Summers—the *metalheads*— are easy going. No restrictions, just beer.
- The Priest doesn't want to be near the *metalheads* nor the Robot.

Intelligent search

Heuristics

Congratulations! You just used your brain to make an intelligent decision:

Intelligent search

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- Assign later the **least-conflicting** people

Intelligent search

Heuristics

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- Assign first the **most-conflicting** individual
- Proceed with the *easy* constraints
- Assign later the **least-conflicting** people

This is a heuristic!

There's no *better* approach

No Free Lunch

However the same 'intelligent decision' can't be applied to *all* problem instances. Each instance is different and some methods are *better* than others depending on many factors specific to each problem. This is called the No Free Lunch theorem.

Can we characterise problems to decide **which method to use**?

Deciding what to do is hard

The Algorithm Selection Problem

Since no better approach exists, then I should use the best method available depending on the problem. This is NP-Complete.

Do I get better results if I **mix heuristics**? Is there a **common pattern** to all my examples so that I can solve them?

Is there really something to learn from my data?

The Computational Learning Theory

The problem of finding a function which accurately characterises a pattern for a certain kind of problem is NP-hard (as is determining if the next block in a shaft is a Diamond Ore in Minecraft). There's no way to know for sure.

But we can select, and mix, and generate, and modify, and see if we come up with a *generalisation*.

General solvers

Hyper-heuristics

This is the idea behind a hyper-heuristic—a general solver which operates over the **heuristic space** instead of the **solution space**.

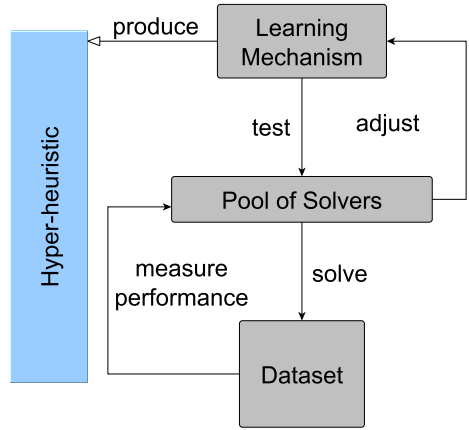
Similar to **algorithm portfolios** and **ensembles**, the hyper-heuristic operates on heuristics and **selects** or **generates** algorithms from existing solvers according to an **objective function**.

The Hyper-heuristic model

Hyper-heuristics

The Learning mechanism could be a **Machine Learning model** or a **metaheuristic**:

- Evolutionary and stochastic algorithms
- Swarm intelligence & Simulated Annealing
- Reinforced & Q-learning
- Neural Networks

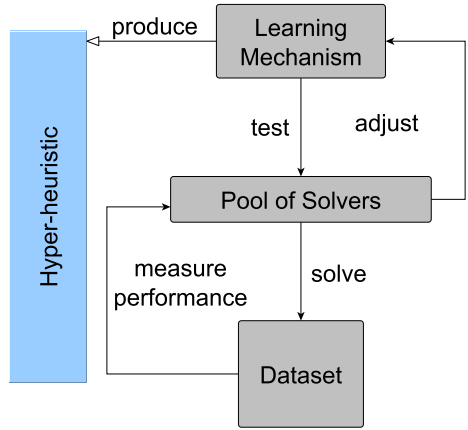


The Hyper-heuristic model

Hyper-heuristics

Both the Pool of available Solvers and the Dataset depend on your **problem domain**:

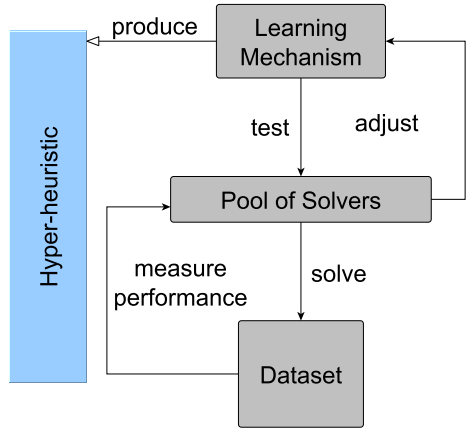
- Knapsack & Bin Packing
- Constraint Satisfaction
- Jobshop Scheduling & Vehicle Routing



The Hyper-heuristic model

Hyper-heuristics

The Performance Measure is usually an **objective function**, per instance or overall (per dataset)

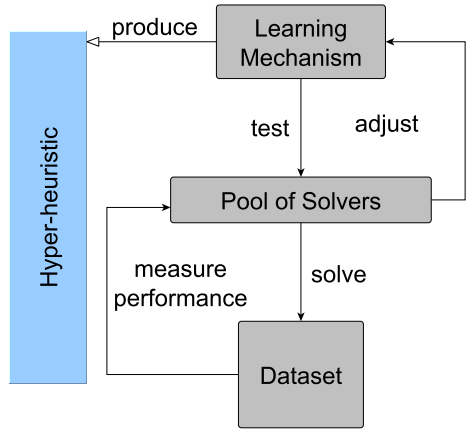


The Hyper-heuristic model

Hyper-heuristics

The output hyper-heuristic could be anything from

- A function (mapping heuristics to instances)
- A vector (or sequence of heuristics)
- A probability distribution (of heuristics)
- A trained model (or pattern description)



It is now a search problem

Hyper-heuristics

Using a **hyper-heuristic** the problem is *solved* by searching for a valid, feasible or a good approximation of the optimal solution.

Therefore, the difficulty now lies on **optimising the search**, which is another hard problem.

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Hyper-heuristics

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Therefore, the difficulty now lies on **optimising the search**, which is another hard problem.

We do our best.

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Thank you!

Slides available at

https://saxarona.github.io/project/hhs-intro/

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